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About the Exactness of the Linear Response Theory

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Abstract. For quantum lattice systems with finite range potentials and integrable space clustering, we prove the linearity of the response theory when dealing with fluctuation observables.

1. Introduction

In statistical mechanics an equilibrium state of a finite system is given by a Gibbs state. In the thermodynamic limit, i.e. for infinite systems, the equilibrium states are described by states satisfying the KMS-condition.

A problem is to understand the occurrence of equilibrium from a dynamical point of view. This is tackled in different ways. One can study the problem by considering the system as part of a larger one. This leads to open system considerations, where topics like the master equation are widely studied (see e.g. [1]). Some aspects of this theory have been made rigorous by several authors in the last decade [2]. Another way to approach the problem is to consider small dynamical perturbations of the system and to observe the effect on the system. Technically one considers a straightforward Dyson expansion of the perturbed dynamics in terms of the unperturbed one. It is often argued that when the perturbation is small and when near to equilibrium, one can limit the study of the response to the first order term in the expansion. This is the basis of the well known "linear response theory" of Kubo [3, 4]. Some aspects of this linear response theory as developed in [3] and [5] have been proved rigorously in [6, 7]. However a lot had to be done to get a complete theory of linear response. This theory consists in a simplistic first order perturbational calculation for which there is not a general theory for treating systematically the higher order terms. On the other hand the linear response actually observed in macroscopic systems has a physical significant range of validity.

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