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How Covariant Closed String Theory Solves a Minimal Area Problem

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Abstract. For a given genus g Riemann surface with $n \geqq 0$ punctures ($n \geqq 3$ for $g=0$) we consider the problem of finding the metric of minimal area under the condition that the length of any nontrivial closed curve be greater or equal to 2π. The minimal area metrics are found for the case of all punctured genus zero surfaces and for many of the higher genus surfaces both with and without punctures. These metrics are induced by Jenkins-Strebel quadratic differentials. They arise from the string diagrams corresponding to restricted Feynman graphs of a closed string field theory action containing classical and quantum restricted polyhedra.

1. Introduction and Summary

During the last year considerable progress was made on the formulation of covariant closed string field theory. The classical field theory turned out fully nonpolynomial and its interactions were determined completely [SaZw, KKS]. It was proven that no purely cubic theory with a symmetric vertex could ever be satisfactory [SoZw 1], and, modulo a technical assumption, it was established that covariant closed string field theory exists to all orders in the loop expansion [SoZw 2]. Finally, the various terms entering into the classical action and the gauge transformation of the string field have been written quite explicitly in [KS].

The determination of the string interactions and string diagrams for the classical closed string field theory was no simple matter. The most useful tool was found to be a theorem of Strebel [St 1, St 2] which was used to understand how to construct Riemann surfaces as contact interactions [SaZw]. This theorem, however, could not be used to understand the surfaces that arise via Feynman diagrams with internal lines. It was nevertheless possible to determine the complete set of interactions [SaZw, KKS] and to give an argument that suggested

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