Commun. Math. Phys. 136, 43–52 (1991)

Communications in Mathematical Physics

Phase Transition in Gas of Hard Core Spheres

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Received July 11, 1989; in revised form July 25, 1990

Abstract. We present a new method of analyzing the gas of hard core spheres. We investigate analytic properties of the thermodynamic function over the circle of convergence of the cluster expansion and describe the way in which phase transition occurs.

1. Introduction

Our aim is to investigate the analytic properties of the thermodynamic function when intensive variables take values on the positive axis not only in the circle of convergence of the Mayer expansion or spectral radius of Kirkwood-Salsburg operator. We use the explicit form of Mayer series coefficients patterned on the cluster expansion [2] and find furthermore their new formulas for the case of hard core gas. We formulate an identity for operator valued generating function and apply it together with an analytic version of Fredholm alternative. Next we explain how the phase transition can occur. We find new regions of analyticity for a thermodynamic function. Our method may be utilized in statistical mechanics and Euclidean field theory.

2. New Form of Mayer Coefficient

In the case when the interaction is a translation-invariant Mayer expansion coefficients have the following form:

$$b_n = \frac{(-\beta)^{n-1}}{n} \sum_{\eta} \int_{[0,1]^{n-1}} d\sigma_{n-1} \int_{\mathbb{R}^{3(n-1)}} d(y)_{n-1} f(\eta, \sigma_{n-1}) \times \prod_{i=1}^{n-1} v(y_{i+1} - y_{\eta(i)}) \exp -\beta W^{(n)}(\sigma_{n-1}) \tag{1}$$