Commun. Math. Phys. 135, 615–640 (1991)

Communications in
Mathematical
Physics

2-*d* Physics and 3-*d* Topology

L. Crane

Department of Mathematics, Yale University, New Haven, CT 06520, USA

Received March 2, 1990

Abstract. Invariants of three dimensional manifolds and of framed oriented labeled links in them are rigorously defined using any solution to the Moore-Seiberg axioms for a Rational Conformal field theory. These invariants are generalizations of Witten's Chern-Simons path integrals. Connections are explored with supersymmetry, four dimensional manifolds, and quantum gravity.

I. Introduction

In [1] E. Witten studied a new object which he called a Chern-Simons path integral. By means of this formal expression, he was able to make a connection between a family of physical systems in two dimensions, the WZW models (which are the classical examples of rational conformal field theories), and a new set of invariants for three dimensional configurations, such as closed manifolds or closed manifolds containing framed links.

Unfortunately, the Chern-Simons path integral does not have a rigorous mathematical definition. Thus Witten's work does not constitute a proof that the expressions he produces are in fact topological invariants.

In [2], I outlined a proof of this in the simplest geometrical situation: a compact oriented 3-manifold with no boundary and no link. My proof avoided any consideration of three dimensional path integrals; and used the axiomatic description of a rational conformal field theory of Moore and Seiberg [3]. Thus, my result is mathematically rigorous, and more general than Witten's statement since it uses any RCFT, but deals with a less general topological configuration.

The key concept which connects rational conformal field theory to topology in three dimensions is duality. Duality is the physical principle which states that a physical process on a surface should be independent of the choice of decomposition for the surface. Changing decompositions of a surface is a process which can describe surface maps and braiding on surfaces; hence it can give information on links and three manifolds.