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Parabolic Methods for the Construction of Spacelike Slices of Prescribed Mean Curvature in Cosmological Spacetimes

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Abstract. Spacelike hypersurfaces of prescribed mean curvature in cosmological spacetimes are constructed as asymptotic limits of a geometric evolution equation. In particular, an alternative, constructive proof is given for the existence of maximal and constant mean curvature slices.

1. Introduction

In recent recent years spacelike hypersurfaces of prescribed mean curvature have played an important role in the study of Lorentzian manifolds. Maximal surfaces, i.e. surfaces of zero mean curvature were used in the first proof of the positive mass theorem [SY 1, SY 2] and in the analysis of the Cauchy problem for asymptotically flat spacetimes [CBY, LA], see also [MT] for further references. Spacelike hypersurfaces of constant mean curvature were studied in [CE, CY, TA]. In [GC] Gerhardt obtained general existence and regularity results for prescribed mean curvature surfaces in cosmological spacetimes and in [B 1] Bartnik settled the corresponding problem for asymptotically flat Lorentzian manifolds, see also [BS] for related results. In these articles the existence proofs are non-constructive as they rely on topological fixed point theorems.

In this paper we use parabolic equations to *construct* spacelike hypersurfaces of prescribed mean curvature in cosmological spacetimes $\mathscr{V} = \Sigma^n \times I$. In this setting a spacelike hypersurface can be written as the graph of a real valued function u over some fixed Cauchy surface Σ^n and the problem of finding a surface of prescribed mean curvature $\mathscr{H}$ reduces to a quasilinear elliptic equation for u. It was shown in [GC] that this equation can be solved under weak assumptions for $\mathscr{H}$ provided the existence of suitable barriers in the timelike future and past is known. In particular, it was proven that in cosmological spacetimes which satisfy the timelike convergence condition and admit a *big bang* and a *big crunch* there exists a foliation of $\mathscr{V}$ by hypersurfaces of constant mean curvature. For each $\mathscr{H} \equiv \tau \neq 0$ there is a unique surface S_τ of constant mean curvature τ and there exists at least