

# $SU(2)$ Chern–Simons Theory at Genus Zero

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**Abstract.** We present a detailed study of the Schrödinger picture space of states in the  $SU(2)$  Chern–Simons topological gauge theory in the simplest geometry. The space coincides with that of the solutions of the chiral Ward identities for the WZW model. We prove that its dimension is given by E. Verlinde’s formulae.

## 1. Introduction

The most characteristic feature of two-dimensional conformal field theories is the separation of left-moving and right-moving degrees of freedom. In the euclidean world, this translates to the factorization of Green functions into sums of products of holomorphic and antiholomorphic expressions: the conformal blocks. In the simplest case of rational conformal theories the sums are finite. Conformal blocks are multivalued and may be naturally viewed as sections of finite-dimensional holomorphic vector bundles over the moduli spaces of punctured Riemann surfaces. The way they are put together to form Green functions is determined by a hermitian metric on the bundles. This point of view was advocated by Friedan and Shenker [3] who formulated conformal field theories in terms of modular analytic geometry. A lot of effort has been invested in analysis of the general structure of Friedan–Shenker bundles, especially in translating the information they encode into an algebraic language [17, 16, 13, 14]. In particular, E. Verlinde’s work, based on the expected factorization properties of the bundles and their modular properties, has allowed to come up with a formula for their dimensions, see also [14].

Among rational conformal field theories, a special role is played by Wess–Zumino–Witten (WZW) sigma model with fields taking values in a compact group  $G$  [18, 10, 6]. They generate through the so-called “coset construction” [7], a rich, possibly exhaustive, family of rational theories. For the WZW models, the Friedan–Shenker bundles are composed of solutions of current-algebra Ward identities. In [19], Witten has observed that they may also be viewed as bundles of Schrödinger-picture quantum states for a three-dimensional non-abelian gauge theory with action given by an integral of the Chern–Simons form. The insertion points of the