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Decay of Two-Point Functions for $(d+1)$-Dimensional Percolation, Ising and Potts Models with d-Dimensional Disorder

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Abstract. Let $\{J_{\langle x,y\rangle}\}_{\langle x,y\rangle\subset\mathbf{Z}^d}$ and $\{K_x\}_{x\in\mathbf{Z}^d}$ be independent sets of nonnegative i.i.d.r.v.'s, $\langle x,y\rangle$ denoting a pair of nearest neighbors in $\mathbf{Z}^d$; let $\beta,\gamma>0$. We consider the random systems: 1. A bond Bernoulli percolation model on $\mathbf{Z}^{d+1}$ with random occupation probabilities

$$q_{(x,t),(y,s)}=\begin{cases}1-e^{-2\beta J_{\langle x,y\rangle}} & \text{if } t=s,\, x,y \text{ nearest neighbors}\\ 1-e^{-2\gamma K_x} & \text{if } x=y,\, |t-s|=1\\ 0 & \text{otherwise}\end{cases}.$$

2. Ferromagnetic random Ising–Potts models on $\mathbf{Z}^{d+1}$; in the Ising case the Hamiltonian is

$$H=-\beta\sum_t\sum_{\langle x,y\rangle}J_{\langle x,y\rangle}\sigma(x,t)\sigma(y,t)-\gamma\sum_x\sum_t K_x\sigma(x,t)\sigma(x,t+1).$$

For such $(d+1)$-dimensional systems with d-dimensional disorder we prove:
(i) for any $d\geqq 1$, if β and γ are small, then, with probability one, the two-point functions decay exponentially in the d-dimensional distance and faster than polynomially in the remaining dimension,
(ii) if $d\geqq 2$, then, with probability one, we have long-range order for either any β with γ sufficiently large or β sufficiently large and any γ.

Introduction

Let $\mathscr{J}=\{J_{\langle x,y\rangle}\}_{\langle x,y\rangle\subset\mathbf{Z}^d}$ and $\mathscr{K}=\{K_x\}_{x\in\mathbf{Z}^d}$ be independent sets of nonnegative

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