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A Derivation of the Broadwell Equation*

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Abstract. We consider a stochastic system of particles in a two dimensional lattice and prove that, under a suitable limit (i.e. $N \to \infty$, $\varepsilon \to 0$, $N\varepsilon^2 \to \text{const}$, where N is the number of particles and ε is the mesh of the lattice) the one-particle distribution function converges to a solution of the two-dimensional Broadwell equation for all times for which the solution (of this equation) exists. Propagation of chaos is also proven.

1. Introduction

The dynamics of the molecules of a rarefied gas could, in principle, be described by Newton's equations, but a reduced description, as provided by the Boltzmann equation, turns out in practice to be more appropriate and useful. On the other hand it is known that in a low density regime, mathematically expressed by the Boltzmann–Grad limit (B–G limit), the solutions of the Newton equations approximate those of the Boltzmann equation. This is rigorously proven for short times in [6] and globally in time for an expanding cloud of gas in the vacuum, [5]. Both cases are based on the fact that the average number of collisions experienced by any given particle is finite and small. At the present moment no other results concerning more general situations are available.

In this paper we deal with a discrete velocity kinetic equation, namely the two dimensional Broadwell equation. There are pathologies connected with the finite structure of the velocity space: from one side it has been proven, [7], that such an equation does not hold in the B–G limit for a natural class of approximating Hamiltonian systems; on the other side, the recent existence theory, [4], for the Boltzmann equation does not apply to discrete velocity models, essentially for the same reason.

In the present paper we prove that the two-dimensional Broadwell equation holds in the B–G limit for a sequence of stochastic particle systems. The

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