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Space-Dependent Dirac Operators and Effective Quantum Field Theory for Fermions*

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Abstract. For the operator $i\partial\!\!\!/ + m(x)$, where $m(x)$ can change sign, we develop a cluster expansion for computing the determinant and Green's functions. We use a local chiral transformation to relate the space-dependent case to the ordinary Dirac operator.

1. Introduction

The study of multi-phase field theories with generalized Yukawa interactions provides a natural structure for studying Dirac operators with space-dependent mass. Different phases of such a model will have different effective fermion masses. If one attempts to analyze such a model via a cluster expansion, different cluster will be in different phases and have different masses for the fermions.

Our specific motivation for studying operators like $i\partial\!\!\!/ + m(x)$ comes from trying to understand the phase structure of two-dimensional Wess–Zumino models. While the single phase case has been studied [15] and much is known for the system in finite volume [9–12], the infinite volume multiphase problem remains unexplored.

A first step to understanding the behavior of the Wess–Zumino model is to study a simpler toy model with almost no bosonic field. By "almost no" field we mean that the only remnant of the boson is a restriction that each block of space-time is in a particular phase. This results in the study of a Dirac operator $i\partial\!\!\!/ + m(x)$ in two dimensions where $m(x)$ takes on a small number of values.

Dirac Operators with Space Dependent Masses. To obtain a view of the technical

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