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Non-Orientable Strings

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Abstract. We describe the halfcomplex structure on non-orientable two-dimensional surfaces and calculate chiral determinants and Laplacians necessary for construction of the Polyakov measure.

Introduction

Most considerations in Polyakov's string theory [1] are confined to the case of two-dimensional compact orientable Riemannian surfaces, or (due to cancellation of conformal anomaly) to one-dimensional compact complex manifolds. The number of works devoted to other logical possibilities, namely to open and non-orientable strings is smaller [2–8]. Partly this is connected with the well-known reduction of open and non-orientable surfaces (so-called Klein surfaces) to orientable and closed ones. More precisely, the category of orientable Riemannian surfaces with antiholomorphic involution is equivalent to the category of Klein surfaces. Factorization of this surface (called double) under the involution gives an open surface if the involution has fixed points and a non-orientable one if it has not. (Open non-orientable strings correspond to orientable surfaces with two involutions.)

In this work, devoted to the non-orientable case only, we tried (when it was possible) to treat all objects (j-differentials, $\bar{\partial}$-operators and so on) without reference to double.

Thinking in this direction we have rediscovered a notion of semicomplex structure (known as dianalytical structure in mathematical works [9]), and have reached a rather unusual generalization of holomorphic bundles. The number of fermion bundles on a non-orientable surface K is two times more than this number for its double X (Sect. 1). An explanation is easy. There is a nontrivial bundle ε on K, called the orientation bundle, which becomes trivial when lifted on X. Fermions on K can be divided in two classes: obtained from the fermion bundle $\mathscr{L}$ on X or having the form $\pi_*\mathscr{L}\otimes\varepsilon$.