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Universal Teichmüller Space and Diff S^1/S^1 *

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Abstract. We point out that the coset space Diff S^1/S^1 is a dense complex submanifold of the Universal Teichmüller Space S of compact Riemann spaces of genus $g \geqq 1$. A holomorphic map of S into the infinite dimensional Segal disk D_1 is constructed. This is the Universal analogue of the map of Teichmüller spaces into the Siegel disk provided by the period matrix. The Kähler potential for the general homogenous metric on Diff S^1/S^1 is computed explicitly using the map into D_1. Some applications to string theory are discussed.

There are many reasons to believe that there is a string theory [1] of quantum gravity. Since classical gravity has a natural formulation in terms of Riemannian geometry, it is reasonable to expect that quantum gravity can be formulated in terms of its complex analogue, Kähler geometry. By combining these two surmises, it is natural to seek a formulation of string theory in terms of Kähler geometry. One approach to this was developed by one of us in collaboration with Bowick and Rajeev [2]. In that approach the basic object of study [1] is the coset space Diff S^1/S^1, which was proved to be a homogenous Kähler manifold. It was shown that this manifold has a finite Ricci tensor (a non-trivial fact in infinite dimensions) which gives a natural explanation of the critical dimension 26 of string theory.

Complex geometry also arises in the conventional perturbative string theory although in a completely different way. The g-loop scattering amplitude of string theory can be expressed as an integral of the square of a holomorphic function on a complex manifold, the Teichmüller space of Riemann surfaces of genus g. The

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1 Throughout this paper Diff S^1 will denote the group of *orientation preserving* diffeomorphisms of the circle