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Spectral Properties of a Tight Binding Hamiltonian with Period Doubling Potential

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Abstract. We study a one dimensional tight binding hamiltonian with a potential given by the period doubling sequence. We prove that its spectrum is purely singular continuous and supported on a Cantor set of zero Lebesgue measure, for all nonzero values of the potential strength. Moreover, we obtain the exact labelling of all spectral gaps and compute their widths asymptotically for small potential strength.

I. Introduction

The discovery of the quasi-crystalline phase in AlMn by Schechtman et al. [1] (see e.g. [2] for a review) has provoked an increasing interest in physical systems that are neither periodic nor random, i.e. neither crystalline nor amorphous. Besides the host of experimental studies, there have been considerable efforts to study theoretically the properties of such systems. Most results are so far confined to the one-dimensional case, most notably to the one dimensional tight-binding hamiltonian

$$H_V = -\Delta + \hat{V}, \tag{1.1}$$

where $\hat{V}$ is a diagonal matrix whose diagonal elements are given by some aperiodic sequence V_n. The problem of studying the spectral properties of such operators turns out to be an extremely interesting mathematical problem in itself, and there is by now a considerable amount of literature, both heuristic and rigorous, dealing with it. There is a considerable amount of general results concerning the case where V_n is quasiperiodic [3, 1, 4]. Among these, the Fibonacci sequence has attracted special attention [5], and it has been proven by Sütö [6] and by Bellissard et al. [7] that, for any nonzero value of the potential strength, the spectrum of H_V in this case