Commun. Math. Phys. 135, 303–312 (1991)

Communications in
Mathematical
Physics

A Discretization of p-Adic Quantum Mechanics

Yannick Meurice*

Departamento de Física, Centro de Investigación y Estudios Avanzados del I.P.N., Apartado Postal 17-740, Mexico 07000, D.F.

Received May 7, 1990

Abstract. We show that some compact subgroups ($\mathscr{H}_{n,m}$) of the p-adic Heisenberg group act irreducibly on corresponding finite dimensional spaces of test-functions ($S_{m,n}$). Under certain conditions, a compact group ($\mathscr{A}_{m+n}$) of linear canonical transformations, isomorphic to $SL(2,\mathbf{Z}_p)$, can be represented unitarily on $S_{m,n}$ as a group of automorphisms of $\mathscr{H}_{n,m}$. The restriction to $S_{m,n}$ can be considered as a discretization because an invariant subgroup ($\mathscr{I}_{m,n}$) of A_{m+n} is represented trivially. It is possible to take a limit where $\mathscr{I}_{m,n}$ becomes an arbitrarily small neighborhood of the identity, while the dimension of $S_{m,n}$ becomes arbitrarily large. This is a possible definition of the "continuum limit" that we relate to other projective limits appearing naturally in the present context.

1. Introduction

In elementary quantum mechanics, the state of a system is represented by a *ray* in a complex vector space [14]. In other words, states differing by a phase (unimodular complex number) are not physically distinguishable. In ray space, the representations of the transformation groups of Hamiltonian mechanics are only required to be representations up to a phase, i.e. projective representations. For instance, translations in positions and momenta commute and form an abelian group; however, the interchange of their representatives in ray space may produce a phase. A well-known example of such a realization is the Heisenberg group.

Let us first consider the usual situation: the positions and momenta are real numbers and the algebra of infinitesimal transformations of the Heisenberg group are the usual commutation relations of quantum mechanics. We now summarize some of the results obtained by C. Itzykson [7]. One can construct a unitary realization of the group of linear canonical transformations as a group of automorphisms of the Heisenberg group. With an appropriate choice of phase, this unitary realization turns out to be a representation up to a sign. In the

* Address after August 1 1990; University of Iowa, Iowa City, Iowa 52242, USA