

Homological Representations of the Hecke Algebra*

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Received January 30, 1990; in revised form April 17, 1990

Abstract. In this paper a topological construction of representations of the $A_n^{(1)}$ -series of Hecke algebras, associated with 2-row Young diagrams will be given. This construction gives the representations in terms of the monodromy representation obtained from a vector bundle on which there is a natural flat connection. The fibres of the vector bundle are homology spaces of configuration spaces of points in \mathbb{C} , with a suitable twisted local coefficient system. It is also shown that there is a close correspondence between this construction and the work of Tsuchiya and Kanie, who constructed Hecke algebra representations from the monodromy of n -point functions in a conformal field theory on \mathbb{P}^1 . This work has significance in relation to the one-variable Jones polynomial, which can be expressed in terms of characters of the Iwahori-Hecke algebras associated with 2-row Young diagrams; it gives rise to a topological description of the Jones polynomial, which will be discussed elsewhere [L2].

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* This work was supported by a SERC studentship grant

** The author is a Lindemann Fellow of the English Speaking Union