Commun. Math. Phys. 135, 91–100 (1990)

Communications in
Mathematical
Physics

Supermoduli Spaces

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Received April 16, 1990

Abstract. The connection between different supermoduli spaces is studied. It is shown that the coincidence of the moduli space of $(1|1)$ dimensional complex manifolds and $N=2$ superconformal moduli space is connected with hidden $N=2$ superconformal symmetry in the superstring theory.

Let W denote the Lie superalgebra of vector fields on $\mathbb{C}^{1,1}$

$$\xi = P(z,\theta)\frac{\partial}{\partial z} + Q(z,\theta)\frac{\partial}{\partial\theta}$$

(here $P(z,\theta)$ and $Q(z,\theta)$ are finite linear combinations of $z^n, z^n\theta, n$ is an integer). It is proved that this Lie algebra is isomorphic to the Lie superalgebra $K(2)$ consisting of $N=2$ infinitesimal superconformal transformations [1, 2]. One can show that this fact is closely related with hidden $N=2$ superconformal symmetry in the superstring theory [3]. For superghost system (and for a general B–C system) hidden $N=2$ supersymmetry was discovered in ref. 4. To understand the origin of the $N=2$ supersymmetry of the B–C system we recall that the fields B and C can be considered as sections of line bundles ω^k and ω^{1-k} correspondingly. However the line bundle ω and its powers can be determined not only for a superconformal manifold but also for arbitrary $(1|1)$ dimensional complex supermanifold M. (If $(\tilde{z},\tilde{\theta})$ and (z,θ) are co-ordinate systems in M, then the transition functions of the line bundle ω^k are equal to D^k, where $D=D(\tilde{z},\tilde{\theta}|z,\theta)$ denotes the superjacobian.)

Let us consider a $(1|1)$ dimensional compact complex supermanifold M, a point $m\in M$ and local complex co-ordinates (z,θ) in the neighbourhood of m. (Here z is even, $|z|\leqq 1$, and θ is odd.) The moduli space of such data will be denoted by $\mathscr{P}$.

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