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Vortices in Holomorphic Line Bundles over Closed Kähler Manifolds

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Abstract. We apply a modified Yang–Mills–Higgs functional to unitary bundles over closed Kähler manifolds and study the equations which govern the global minima. The solutions represent vortices in holomorphic bundles and are direct analogs of the vortices over $\mathbf{R}^2$. We obtain a complete description of the moduli space of these new vortices where the bundle is of rank one. The description is in terms of a class of divisors in the base manifold. There is also a dependence on a real valued parameter which can be attributed to the compactness of the base manifold.

Introduction

Many interesting equations in gauge theory arise as minimizing conditions for gauge invariant functionals. The self- and anti-self dual Yang–Mills equations, the Hermitian–Einstein equation, the Bogomoln'yi monopole equations and the vortex equations are all equations of this sort (cf. [A–H–S], [F–U], [J–T]). Much of the interest in such equations lies in the conditions for existence of solutions and in the moduli space of gauge equivalence classes of solutions (cf. [A–H–D–M], [D1], [D2], [Hi], [H–M], [T]). In this paper we describe a new addition to this collection of "minimizing equations" and address the question concerning the moduli space of its solutions.

In the case of the self- and anti-self dual equations, the functional being minimized is the Yang–Mills functional over $\mathbf{R}^4$. The solutions to these equations form a special class of connections on principal bundles over $\mathbf{R}^4$. All such solutions have associated to them an integer valued "quantum number" known as the instanton number. Both the existence of the special equations for the extrema and the explanation of the instanton numbers can be attributed to the same thing; the key point is that in four dimensions the Yang–Mills functional can be split as a

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