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The Quantum Group Structure Associated with Non-Linearly Extended Virasoro Algebras

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Abstract. Recently, an infinite family of chiral Virasoro vertex operators, whose exchange algebra is given by the universal R-matrix of $SL(2)_q$, has been constructed. In the present paper, the case of non-linearly (W-) extended Virasoro symmetries, related to the algebras A_N, $N>1$, is considered along the same line. Contrary to the previous case (A_1) the standard R-matrix of $SL(N+1)_q$ does not come out, and a different solution of Yang and Baxter's equations is derived. The associated quantum group structure is displayed.

Quantum groups seem to underlie the very basic structure of conformal field theories. In particular, for the Virasoro family (2D gravity and minimal models) a precise connection has just been established [1, 2]. The method is to relate the monodromy properties of the differential equation satisfied by the chiral vertex operators to the R-matrix of $SL(2)_q$. The first step in this direction was taken by Babelon [1] who established such a relationship for the simplest $J=1/2$ representation. This was fully generalized by one of us (J.-L. G.) [2] who recently constructed an infinite family of chiral vertex operators whose exchange algebra is given by the universal R-matrix of $SL(2)_q$. This Virasoro family is intrinsically related to the simple algebra A_1, since it may be generated from Liouville theory which is the A_1 Toda field theory [3], and includes the $(SU(2)_k \otimes SU(2)_1)/SU(2)_{k+1}$ coset models. There are more general conformal families of the same type [3–5] with non-linearly (W-) extended Virasoro algebras, which are associated with the higher simple Lie algebras [3, 5]. In the present paper, we begin the extension of the construction carried out in [1, 2] to these models, by deriving an exchange algebra for the simplest set of chiral operators (in the defining representation) similar to the spin 1/2 case worked out in [1]. It will be shown that contrary to the Virasoro family, the standard R-matrix of the associated quantum group does not

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