Commun. Math. Phys. 134, 587–617 (1990)

Communications in
Mathematical
Physics

Similarity Between the Mandelbrot Set and Julia Sets

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Received July 10, 1989

Abstract. The Mandelbrot set M is "self-similar" about any Misiurewicz point c in the sense that if we examine a neighborhood of c in M with a very powerful microscope, and then increase the magnification by a carefully chosen factor, the picture will be unchanged except for a rotation. The corresponding Julia set J_c is also "self-similar" in the same sense, with the same magnification factor. Moreover, the two sets M and J_c are "similar" in the sense that if we use a very powerful microscope to look at M and J_c, both focused at c, the structures we see look like very much the same.

1. Introduction

For a quadratic polynomial $f_c: z \mapsto z^2 + c$, the filled-in Julia set K_c of f_c is the set of non-escaping points under iteration:

$$K_c = \{z \in \mathbb{C} \mid (f_c^n(z))_{n \in \mathbb{N}}, \text{ is bounded}\},$$

where f_c^n denotes the n^{th} iteration $f_c \circ f_c \circ \ldots \circ f_c$ of f. The Julia set of f_c is $J_c = \partial K_c$.

The Mandelbrot set is

$$M = \{c \in \mathbb{C} \mid 0 \in K_c\}.$$

One can generate easily the pictures of Julia sets and the Mandelbrot set by computers. Figure 1 is a picture of M, Fig. 3a–3d are pictures of J_c for various values of c. Globally, J_c and M have completely different shapes. However, their local structures are sometimes very similar. Figure 2 consists of three successive enlargements of M in a neighborhood of i. A remarkable resemblance with the Julia set for $c = i$ (Fig. 3a) appears. In fact, this kind of similarity happens for every value of c which is a Misiurewicz point, that is, for which the point 0 under f_c is not

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