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Periodic and Flat Irreducible Representations of $SU(3)_q$

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Abstract. We construct all the periodic irreducible representations of $\mathscr{U}(SU(3))_q$ for q a m-root of unity. Their dimensions are $k(2m)^2$ for $k = 1, \ldots, m$ $\left(\text{only } k = 1, \ldots, \frac{m}{2} \text{ for even } m\right)$. Their interest is that they could be a tool to generalize the chiral Potts model. By truncation of these representations, we construct "flat representations" of $\mathscr{U}(SU(3))_q$, in which all the multiplicities of the weights are set to 1.

I. Introduction

In [1], M. Rosso classified the finite dimensional irreducible representations of the quantum analogue $\mathscr{U}(\mathscr{G})_q$ of the enveloping algebra of a complex simple Lie algebra when the parameter of deformation q is not a root of unity. He proved that they were deformations of the finite dimensional irreducible representations of the classical $\mathscr{U}(\mathscr{G})$. They are in particular characterized by a highest weight λ corresponding to a classical representation of $\mathscr{U}(\mathscr{G})$ and by $\omega \in \{1, -1, i, -i\}$ characterizing the average (the center value) of the eigenvalues of the generators h_i of the Cartan torus.

In [2], the finite dimensional irreducible representations of $\mathscr{U}(SU(2))_q$ for q a root of unity are classified. The new fact is that the dimensions of these representations is bounded by m, if $q^m = 1$. The $d < m$ representations are called regular and correspond to unitary representations of the WZW theory based on affine $SU(2)$ level $m - 2$. Furthermore, the m-dimensional irreducible representations can be periodic, in the sense that the generators J^+ and J^- are not nilpotent and act as $\mathbb{Z}_m$. Continuous parameters also enter in their definition. In [3], the composition of regular representations is studied. It leads to a sum of irreducible and indecomposable representations, an explicit truncation being possible to recover the sum over regular representations provided by the WZW theory. This result is generalized in [4] to all the quantum analogues of simple Lie algebra.