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The Structure of the W_∞ Algebra

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Abstract. We prove rigorously that the structure constants of the leading (highest spin) linear terms in the commutation relations of the conformal chiral operator algebra W_∞ are identical to those of the $\mathrm{Diff}_0^+ \mathbb{R}^2$ algebra generated by area preserving diffeomorphisms of the plane. Moreover, all quadratic terms of the W_N algebra are found to be absent in the limit $N \to \infty$. In particular we show that W_∞ is a central extension of $\mathrm{Diff}_0^+ \mathbb{R}^2$ with non-trivial cocycles appearing only in the commutation relations of its Virasoro subalgebra. We also propose a representation of W_∞ in terms of a single scalar field in $2+1$ dimensions and discuss its significance in the context of quantum field theory.

1. Introduction

The construction of all unitary highest weight representations of the infinite dimensional symmetry algebras that arise in two dimensional conformal field theory has provided a non-perturbative framework for solving a large class of physically interesting quantum field theory models (see for instance [1] and references therein). One of the most striking results in the classification of rational conformal field theories was the realization that simple Lie algebras determine the structure and the operator content of unitary scale invariant 2-dim systems. In the chiral operator approach, rational conformal field theories are described as minimal models of extended conformal symmetry algebras $\mathscr{W}$ generated by the stress-energy tensor $T(z)$ and other holomorphic fields $\{w_s(z), s \in J\}$, which are associated with additional conserved currents in the 2-dim world. Typically, the generators of $\mathscr{W}$-algebras are labeled by the vertices of Dynkin diagrams of simple Lie algebras G, which also determine the conformal weight (spin) s of the chiral fields $w_s(z)$. The Virasoro algebra

$$[T(z), T(z')] = (T(z) + T(z'))\delta_{,z}(z - z') + \frac{c}{12}\delta_{,zzz}(z - z') \tag{1}$$

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