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Dimensional Regularization and Renormalization of QED

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Abstract. We give an x-space definition of dimensional regularization suited to the tree expansion method of renormalization. We apply the dimensionally regularized tree expansion to QED, obtaining sharp bounds on the size of a renormalized graph. Subtractions are made with the Lagrangian counterterms of the tree expansion, not by minimal subtraction techniques, and so do not entail a knowledge of the meromorphic structure of a graph as a function of dimension. This renormalization procedure respects the Ward identities, and the counterterms required are gauge invariant.

1. Introduction

In [1] with J. Feldman and T. Hurd, we developed a general scheme for renormalizing a quantum field theory based on the tree expansion of G. Gallavotti and F. Nicolò [2], and we applied this scheme to quantum electrodynamics (QED) to give a complete proof of the renormalizability of QED in perturbation theory. The basic idea of the tree expansion approach is to slice up each field as a sum of fields of different scales, to integrate out the fields one scale at a time, and to renormalize scale by scale. The resulting renormalization procedure is remarkably simple: one never sees "overlapping divergences" or the usual combinatoric complexities of BPHZ renormalization, and the required bounds amount to little more than superficial power counting. We briefly review the tree expansion in Sect. 2 but shall rely on [1] or [3] for details. See also Hurd [4] for a simple version of the tree expansion that employs continuous rather than discrete slicing, as in Polchinski [5].

The main technical difficulty we faced in applying the tree expansion to QED in [1] is that the slicing breaks gauge invariance and so it was not clear whether the theory could be renormalized using only gauge invariant counterterms. We overcame this problem as follows: we introduced an auxiliary regularization on the fermions that preserved the Ward identities but allowed us to remove the tree