Quantum and Classical Pseudogroups. Part II
Differential and Symplectic Pseudogroups

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Abstract. The category of sympletic pseudospaces (analogical to the category of pseudospaces in the sense of [2]) is introduced and used to define symplectic pseudogroups (structures analogical to pseudogroups [3] or quantum groups [4]). It is shown that symplectic pseudogroups are in one-to-one correspondence with Manin groups, also introduced in this paper. The set-theoretical part of these structures has been described in [I].

Introduction

Symplectic pseudogroups introduced in this paper (Sect. 7) are classical (symplectic) counterparts of quantum (pseudo-) groups ([3, 4], ...). They play in classical theory the same role as quantum groups in quantum theory. They also seem to be useful for constructing quantum groups.

Symplectic pseudogroups are symplectic manifolds with a structure similar to Hopf (or Kac) algebra, expressed in terms of symplectic relations (multiplication, unit, inverse, comultiplication, etc.).

If we neglect the symplectic and differential structure of the underlying manifold, our symplectic pseudogroup becomes a union pseudogroup. Union pseudogroups have been introduced in the first part of this paper which we refer to as to [I]. The study of union pseudogroups in [I] has to be considered as a first step in our study of symplectic pseudogroups, in which we have separated purely set-theoretical problems from differential- and symplectic-geometrical ones.

Our definition fits in a general scheme of enlarging the category of groups to a self-dual category. A passage to new kind of objects consists in replacing the usual space by a "noncommutative space." In the case of quantum (pseudo-) groups, "noncommutative spaces" are quantum (pseudo-) spaces, i.e. objects dual to C^*-algebras. In the case of symplectic pseudogroups, "noncommutative spaces" are symplectic pseudospaces, i.e. objects dual to S^*-algebras defined in Sect. 3. With morphisms defined in Sect. 4, S^*-algebras form a category which we consider