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Absence of Ballistic Motion

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Abstract. For large classes of Schrödinger operators and Jacobi matrices we prove that if h has only one point spectrum then for ϕ_0 of compact support

$$\lim_{t\to\infty} t^{-2} \|xe^{-ith}\phi_0\|^2 = 0 .$$

1. Introduction

Consider a free Schrödinger particle. Then the Heisenberg position operators obeys

$$x(t) = x + tp$$

since p is a constant of the motion. Thus $|x(t)|$ grows linearly in t, indeed for any $\phi \in \mathscr{S}(\mathbb{R}^n)$:

$$\lim(\phi, x(t)^2\phi)/t^2 = (\phi, p^2\phi) > 0 .$$

This paper had its root in a question of Joel Lebowitz asking if such ballistic motion didn't have its roots in absolutely continuous spectrum. Alas, while it is likely that Joel is correct, I have been able to obtain only partial results. Here I will prove that for Hamiltonians with pure point spectrum (think of the random case [1]), we have that for a dense set of initial ϕ that $(\phi, x(t)^2\phi)/t^2 \to 0$. Unfortunately, I have nothing to say in the singular continuous case.

For background note that it is a result of Radin-Simon [2] that when ϕ is in C_0^∞, $(\phi, x(t)^2/t^2$ is bounded at infinity in great generality.

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