

Conformal Field Algebras with Quantum Symmetry from the Theory of Superselection Sectors

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Abstract. According to the theory of superselection sectors of Doplicher, Haag, and Roberts, field operators which make transitions between different superselection sectors – i.e. different irreducible representations of the observable algebra – are to be constructed by adjoining localized endomorphisms to the algebra of local observables. We find the relevant endomorphisms of the chiral algebra of observables in the minimal conformal model with central charge $c = \frac{1}{2}$ (Ising model). We show by explicit and elementary construction how they determine a representation of the braid group B_∞ which is associated with a Temperley-Lieb-Jones algebra. We recover fusion rules, and compute the quantum dimensions of the superselection sectors. We exhibit a field algebra which is quantum group covariant and acts in the Hilbert space of physical states. It obeys local braid relations in an appropriate weak sense.

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