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Unitary Dressing Transformations and Exponential Decay Below Threshold for Quantum Spin Systems. Parts I and II

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Abstract. We consider a class of quantum spin systems defined on connected graphs of which the following Heisenberg XY-model with a variable magnetic field gives an example:

$$H_\lambda = \sum_{x \in \mathbb{Z}^d} h_x \sigma_x^{(3)} + \lambda \sum_{\langle x,y \rangle \subset \mathbb{Z}^d} (\sigma_x^{(1)} \sigma_y^{(1)} + \sigma_x^{(2)} \sigma_y^{(2)}).$$

We treat first the case in which $h_x = \pm 1$ for all sites x and we introduce a unitary dressing transformation to control the spectrum for λ small. Then, we consider a situation in which $|h_x|$ can be less than one for x in a finite set $\mathscr{S}$ and prove exponential decay away from $\mathscr{S}$ of dressed eigenfunctions with energy below the one-quasiparticle threshold. If the ground state is separated by a finite gap from the rest of the spectrum, this result can be strengthened and one can compute a second unitary transformation that makes the ground state of compact support. Finally, a case in which the singular set $\mathscr{S}$ is of finite density, is considered. The main technical tools we use are decay estimates on dressed Green's functions and variational inequalities.

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