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Multiple Resonances in the Semi-Classical Limit

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Abstract. We construct for the Schrödinger operator in the semi-classical limit compact perturbations of a radial symmetric potential which give rise to resonances associated to arbitrarily high order poles for the meromorphic extension of the resolvent. Our results concern the hamiltonian $P_0 = -h^2\Delta - x^2$ in the 2-dimensional case, as well as a fairly large class of radial-symmetric potentials in the 3-dimensional case. We show that the poles of the resolvent for such a potential are necessarily simple, and subsequently the degeneracy is due to a lack of symmetry.

Introduction

We consider the Schrödinger operator in semi-classical limit on $L^2(\mathbb{R}^n)$:

$$P = -h^2\Delta + V(x) \quad (h \to 0).$$

If $V \in \mathscr{C}^\infty(\mathbb{R}^n : \mathbb{R})$ satisfies:

$$\lim_{|x| \to \infty} V(x) = E_0 > -\infty,$$

then P is essentially self-adjoint on $\mathscr{C}_0^\infty(\mathbb{R}^n)$ as an unbounded operator, and $L^2(\mathbb{R}^n) = \mathscr{H}_{pp} \oplus \mathscr{H}_{ess}$, where $\mathscr{H}_{pp}$ is the sum of the bound state corresponding to the pure point spectrum $\sigma_{pp}(P)$ and $\mathscr{H}_{ess}$ the space of free states associated to the essential spectrum $\sigma_{ess}(P)$. Here the essential spectrum coincides with the absolutely continuous spectrum. The threshold E_0 is the bottom of the essential spectrum

$$\inf \sigma_{ess}(P) = E_0 .$$

In particular, if $E_0 > 0$, there is only discrete spectrum near 0 and for $z \in \mathbb{C}$ small enough, the resolvent $R(z) = (P - z)^{-1}$ is a meromorphic function whose poles (necessarily simple) are the (real) eigenvalues of P.