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Torsion Constraints in Supergeometry

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Abstract. We derive the torsion constraints for superspace versions of supergravity theories by means of the theory of G-structures. We also discuss superconformal geometry and superKähler geometry.

I. Introduction

Supersymmetry is a now well established topic in quantum field theory [WB, GGRS]. The basic idea is that one can construct actions in ordinary spacetime which involve both even commuting fields and odd anticommuting fields, with a symmetry which mixes the two types of fields. These actions can then be interpreted as arising from actions in a superspace with both even and odd coordinates, upon doing a partial integration over the odd coordinates. A mathematical framework to handle the differential topology of supermanifolds, manifolds with even and odd coordinates, was developed by Berezin, Kostant and others. A very readable account of this theory is given in the book of Manin [Ma].

The right notion of differential geometry for supermanifolds is less clear. Such a geometry is necessary in order to write supergravity theories in superspace. One could construct a supergeometry by $\mathbb{Z}_2$ grading what one usually does in (pseudo) Riemannian geometry, to have supermetrics, super Levi-Cività connections, etc. The local frame group which would take the place of the orthogonal group in standard geometry would be the orthosymplectic group. However, it turns out that this would be physically undesirable. Such a program would give more fields than one needs for a minimal supergravity theory, i.e. the fields would give a reducible representation of the superLorentz group. In order to get around this problem, the approach of Wess and Zumino [WZ] is to use the standard orthogonal group as the

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