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7-Dimensional Compact Riemannian Manifolds with Killing Spinors

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Abstract. Using a link between Einstein-Sasakian structures and Killing spinors we prove a general construction principle of odd-dimensional Riemannian manifolds with real Killing spinors. In dimension $n=7$ we classify all compact Riemannian manifolds with two or three Killing spinors. Finally we classify non-flat 7-dimensional Riemannian manifolds with parallel spinor fields.

1. Introduction

A Killing spinor on a Riemannian spin manifold (M^n, g) is a section ψ of the spin bundle S satisfying, for any vector field X, the differential equation

$$\nabla_X \psi = \lambda X \psi ,$$

where $\lambda \neq 0$ is a constant and $X\psi$ denotes the Clifford multiplication of the vector X by the spinor ψ. Solutions of this equation occur quite naturally in Differential Geometry as well as in Mathematical Physics. For example, on a compact Riemannian spin manifold with non-negative scalar curvature R there is a lower bound involving R for the first eigenvalue of the Dirac operator, and eigenspinors to this lower bound are Killing spinors (see [8, 19, 30]). Furthermore, Killing spinors are special solutions of the so-called twistor equation and in case of a compact manifold they generate – up to a conformal change of the metric – all solutions of the twistor equation (see [31]).

The construction of models in supergravity depends on Riemannian manifolds with Killing spinors. There are several papers (see for example [6, 32]) investigating, from this point of view, the properties of Riemannian manifolds with Killing spinors as well as containing the construction of examples.

The existence of a Killing spinor imposes algebraic conditions on the Weyl tensor of the space (see [9]) and on the covariant derivative of the curvature tensor; in particular M^n must be an Einstein space (see [8]). The constant λ is given by

$$\lambda^2 = \frac{1}{4} \frac{R}{n(n-1)} .$$