

# Relativistic Toda Systems★

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**Abstract.** We present and study Poincaré-invariant generalizations of the Galilei-invariant Toda systems. The classical nonperiodic systems are solved by means of an explicit action-angle transformation.

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## 1. Introduction

In recent years it has been shown that the well-known Galilei-invariant Calogero–Moser  $N$ -particle systems admit Poincaré-invariant generalizations. These relativistic particle systems are not only completely integrable at the classical level, but can also be quantized in such a fashion that integrability survives [1, 2]. In this paper we show that relativistic integrable generalizations of the non-relativistic Toda systems [3–5] exist, too. Moreover, we solve the nonperiodic classical systems by constructing an explicit action-angle transformation.

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