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Rate of Escape of Some Chaotic Julia Sets

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Abstract. We give a formula for the rates of escape for Julia sets with pre-periodic critical points and for C^∞ endomorphisms of the interval with non-flat pre-periodic critical points outside the basin of attracting periodic points.

0. Introduction

Let $f : M \hookleftarrow$ be a continuous mapping of a riemannian manifold and $U \subseteq M$. The *rate of escape* of U (by f) is defined to be

$$R(U) := \lim_{n\to+\infty} \frac{1}{n} \log \mathrm{vol}\left(\bigcap_{k=0}^{n-1} f^{-k}(U) \right) \leq 0$$

if this limit exists, i.e. the exponential decay of the volume of the set of points which stay on U for n iterates. If f is Axiom A and U a small neighbourhood of a basic set Λ, Bowen and Ruelle [2], [3] proved that

$$R(U) = P(\varphi^u) = \sup\{h_\nu(f) - \Sigma\lambda_i^+(\nu) \mid \nu \text{ ergodic measure with } \mathrm{Supp}(\nu) \subset \Lambda\},$$

where $\lambda_i^+(\nu)$ are the positive Lyapunov exponents of ν, P is the topological pressure, $\varphi^u(x) = -\log |\det Df|E^u(x)|$ and $E^u(x)$ is the unstable space at $x \in \Lambda$. Axiom A attractors are characterized by $P(\varphi^u) = 0$, when Λ is not an attractor, $P(\varphi^u) = R(U) < 0$ give a measure of the influence of Λ on neighbouring orbits. Similar methods [12] apply to prove that $R(U) = P(\varphi)$, $\varphi(x) = -\log |\det f'(x)|$ if U is a small neighbourhood of K and $f : K \hookleftarrow$ is strictly expanding.

Eckmann and Ruelle [3] raised the conjecture that for *some* open set $U \supset \mathrm{Supp}(\mu)$,

$$R(U) = h_\mu(f) - \Sigma\lambda_i^+(\mu)$$

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