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Multichannel Nonlinear Scattering for Nonintegrable Equations* **

A. Soffer[1] *** and M. I. Weinstein[2]

[1] Department of Mathematics, Princeton University, Princeton: NJ 08544, USA
[2] Department of Mathematics, University of Michigan, Ann Arbor, MI 48109, USA

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Abstract. We consider a class of nonlinear Schrödinger equations (conservative and dispersive systems) with localized and dispersive solutions. We obtain a class of initial conditions, for which the asymptotic behavior ($t \to \pm\infty$) of solutions is given by a linear combination of nonlinear bound state (time periodic and spatially localized solution) of the equation and a purely dispersive part (decaying to zero with time at the free dispersion rate). We also obtain a result of *asymptotic stability* type: given data near a nonlinear bound state of the system, there is a nonlinear bound state of nearby energy and phase, such that the difference between the solution (adjusted by a phase) and the latter disperses to zero. It turns out that in general, the time-period (and energy) of the localized part is different for $t \to +\infty$ from that for $t \to -\infty$. Moreover the solution acquires an extra constant asymptotic phase $e^{i\gamma^{\pm}}$.

1. Introduction

This paper deals with the scattering theory of a class of conservative nonlinear dispersive equations admitting more than one channel. By this we mean that the asymptotic behavior is given by a linear combination of a localized (in space), periodic (in time) wave (solitary or standing wave) and a dispersive part. For nonlinear flows which are completely integrable (e.g. one-dimensional cubic nonlinear Schrödinger, Korteweg-de Vries equations), some analysis of the asymptotic system of, for example, localized part (solitons) plus dispersion can be carried out using the inverse scattering transform [G–G–K–M, Z–S, Lax, C–K]. The inverse scattering transform decouples the localized from the dispersive part.

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