Commun. Math. Phys. 132, 253–283 (1990)

Communications in
Mathematical
Physics

Hydrodynamics of Stationary Non-Equilibrium States for Some Stochastic Lattice Gas Models*

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Dedicated to Res Jost and Arthur Wightman

Abstract. We consider discrete lattice gas models in a finite interval with stochastic jump dynamics in the interior, which conserve the particle number, and with stochastic dynamics at the boundaries chosen to model infinite particle reservoirs at fixed chemical potentials. The unique stationary measures of these processes support a steady particle current from the reservoir of higher chemical potential into the lower and are non-reversible. We study the structure of the stationary measure in the hydrodynamic limit, as the microscopic lattice size goes to infinity. In particular, we prove as a law of large numbers that the empirical density field converges to a deterministic limit which is the solution of the stationary transport equation and the empirical current converges to the deterministic limit given by Fick's law.

1. Introduction

As a common experience, the large scale properties of a system in a non-equilibrium steady state are determined by the stationary solution of the relevant macroscopic equation with appropriate boundary conditions. Just to recall a familiar example: Let us consider a Rayleigh-Bénard cell consisting of a liquid between two plates at different temperatures, T_1 and T_2. The temperature difference is assumed to be sufficiently small so that heat is transported only diffusively and that the velocity field vanishes. In such a situation the hydrodynamic equations have a unique stationary solution with density $\varrho(z)$, velocity $\mathbf{v}=0$ and temperature $T(z)$, $0 \leqq z \leqq h$, $T(0)=T_1$ and $T(h)=T_2$, where z is the direction of the temperature gradient.

From a microscopic point of view we may model the liquid as a collection of a huge number of hard spheres (with a diameter of 1 Å, say), whose time evolution is governed by Newton's equation of motion. Within this framework, the steady state is described by a probability *measure* on phase space. In principle, we know how

* Supported in part by NSF Grants DMR 89-18903 and INT 8521407. H.S. also supported by the Deutsche Forschungsgemeinschaft