Commun. Math. Phys. 132, 189–199 (1990)

Communications in Mathematical Physics

Translation Group and Modular Automorphisms for Local Regions

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Dedicated to Res Jost and Arthur Wightman

I. Introduction

In this paper we will look at quantum field theory in the setting of Araki, Haag, and Kastler. The system of local observables will be denoted by $\{\mathscr{A}(O), \mathscr{A}, \alpha, \mathbf{R}^d\}$. For details see Sect. II. We are interested in positive energy representations, denoted by $\{\pi, U(a), V^+, \mathscr{H}\}$. In this situation every vector analytic for the energy has the Reeh Schlieder [9] property, which means that this vector is cyclic and separating for the algebras $\pi(\mathscr{A}(O))$ whenever O is a bounded region. As a consequence we find that there exists a modular automorphism σ_t associated to every pair $\{\pi(\mathscr{A}(O))'', \Omega\}$.

Inspired by statistical mechanical examples one might think that the generators of the modular automorphism group are connected with the energy belonging to the region we consider. On first sight, however, there seems to be no connection between the space-time-translations and the modular automorphisms. Therefore, Buchholz and Junglas [5] have used the Hamiltonian of the vacuum sector in order to construct K.M.S. states for the theory of local observables. In order to obtain a kind of local energy H^O such that $\exp\{-\beta H^O\}$ is trace class they had to work with the nuclearity condition of Buchholz and Wichmann [6]. Although this local Hamilton operator had some properties similar to the modular operator, they failed to establish a connection between these objects. This is because their "local" Hamiltonian does not generate a group of automorphisms for the local region.

Recently Buchholz, D'Antoni, and Longo [4] found estimates suggesting that, after rescaling, the modular operator of the double cone tends to the Hamilton operator if the double cone tends to the whole space. In the last section we will discuss a possible procedure proving this result.

We want to show that the space-time translations and the modular automorphisms, indeed, are closely related. Using the Malgrange Zerner theorem it easily shows that expressions of the form

$$(\Omega, \alpha_a(A)\sigma_t(B)) =: E(a,t)$$

are the boundary value of an analytic function holomorphic in the variables (a, t) provided α_a represents the space-time translations, σ_t the modular automorphisms,