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Asymptotic Completeness for N-Body Short-Range Quantum Systems: A New Proof

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Dedicated to Res Jost and Arthur Wightman

Abstract. We give an alternative geometrical proof of asymptotic completeness for an arbitrary number of quantum particles interacting through short-range pair potentials. It relies on an estimate showing that the intercluster motion concentrates asymptotically on classical trajectories.

Table of Contents

1. Introduction

The first task of quantum scattering theory is to give a classification of the possible large time behaviours of Schrödinger orbits $e^{-itH}\psi$. In this paper we study this problem for an arbitrary number of particles interacting via short range interactions. In the intuitive picture of the scattering process, this system is well described at large times by a number of bound clusters which do not feel each other. This statement is called asymptotic completeness. For $N=2, 3$ it was proved by several authors (see [2, Sect. 5.7] for a review), and in particular using geometric ideas by Enss [6–8]. For arbitrary N the proof is due to Sigal and Soffer [22].

Our main intermediate result is a propagation estimate showing that asymptotically $2p_a \approx x_a/t$ on that part of configuration space, which corresponds to a given cluster decomposition a. Here we have used the notation of [22] which is reviewed at the end of this section. Such a property in fact is typical for the clusters of a moving freely. This result is not new, since it was derived earlier in [23] (with the only modification that the partition of unity used there is in phase space).