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A Feynman-Kac Formula for the Quantum Heisenberg Ferromagnet. II

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Dedicated to Res Jost and Arthur Wightman

Abstract. This article continues the analysis of the first arcticle under the same title. Using methods of stochastic analysis we prove Feynman-Kac formulas for the relevant heat kernels. We also present classical limit theorems.

This paper is the second part of a work devoted to a probabilistic approach for the quantum Heisenberg ferromagnet relating this model to a Euclidean lattice field theory.

In Sect. 2 and 3 of the previous article heat kernel representations of the partition function were given. In Sect. 4 the resulting Euclidean field theoretic Lagrangian was calculated. Here, in Sect. 5 and 6, we formulate Feynman-Kac representations for the heat kernels involved, first for the one-lattice point theory and then for the full interacting theory on an arbitrary finite lattice. Our presentation is strongly influenced by Bismut's work on probabilistic proofs of index theorems [Bi2].

In Sect. 7 we present classical limit theorems for the purely bosonic sector of the theory.

We use the notations and results of [HMPS].

5. Feynman-Kac Formula for the One Lattice Point Theory

In this section we will establish a rigorous stochastic expression for the kernel of the semigroup

$$e^{-t\{\frac{1}{m}\bar{\square}-id\pi^\lambda(h)\}}, \tag{5.1}$$

where $t>0$ and $h\in\mathbf{g}$. For simplicity we consider the case $m=1$. The general case $m>0$ can be obtained by the rescaling $t\to\frac{t}{m}$, $h\to hm$. The first step is to construct the stochastic process on $\bar{\Lambda}(\mathscr{L}^\lambda)$ that is generated by the (horizontal) Bochner Laplacian $-(\nabla^\lambda)^2$. The stochastic representation of the kernel (5.1) will then be