

$N = 2$ Supergravity, Type IIB Superstrings, and Algebraic Geometry

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Abstract. The geometry of $N = 2$ supergravity is related to the variations of Hodge structure for “formal” Calabi–Yau spaces. All known results in this branch of algebraic geometry are easily recovered from supersymmetry arguments. This identification has a physical meaning for a type IIB superstring compactified on a Calabi–Yau 3-fold. We give exact (non-perturbative) results for the string effective Lagrangian. Our geometrical framework suggests a re-formulation of the Gepner conjecture about $(2, 2)$ superconformal theories as the solution to the *Schottky problem* for algebraic complex manifolds having trivial canonical bundle.

I. Introduction

It is well known that some classical problems in mathematics can be studied using ideas arising from supersymmetry [1]. A partial list of results contains the Witten formulation [2] of Morse theory, the simple proof of the index theorems [3] given by Alvarez–Gaume [4], and the theory of elliptic genera [5].

On the physical side, the understanding of the mathematical implications of susy is also crucial, since it may give non-perturbative insight on the theory [6].

These results arise from *rigid* supersymmetry. It is an easy guess that *local* supersymmetry (supergravity) should be even more powerful as a “mathematical trick.” What is not easy to figure out, is what are the mathematical problems related to supergravity.

In this paper we give a partial answer to this question for $N = 2$ supergravity. We show that this theory is deeply connected with the period maps [7] for algebraic complex threefolds having trivial canonical bundle (Calabi–Yau spaces [8]). Everything is known [9, 10] about these maps, turns out to be a well known fact of supergravity. $N = 2$ SUGRA is a “physicist’s” approach to the theory of Hodge structures and Hodge bundles over moduli spaces.

To give a rough idea of the period map and its relevance for physics, let us consider the situation where its meaning is obvious. Consider the compactification [11] of the type IIB superstring [12] on a Calabi–Yau (CY) 3-fold X . The $4D$