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A Feynman-Kac Formula for the Quantum Heisenberg Ferromagnet. I

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Abstract. The Hamiltonian of the (anisotropic) quantum Heisenberg (anti-) ferromagnet on an arbitrary finite lattice is lifted to a Hamiltonian acting on sections of the bundle obtained by twisting a certain line bundle over the classical spin configuration space (which is a Kähler manifold) with the Dolbeault complex. This procedure is extended from $SU(2)$ to arbitrary compact semi-simple Lie groups and arbitrary irreducible representations. The Bott-Borel-Weil theorem gives a heat kernel representation for the original partition function in an external magnetic field. The $U(1)$-gauged local Hamiltonian is the sum of the free, supersymmetric, twisted Dolbeault Laplace operator (multiplied by the inverse of an arbitrary small mass parameter) plus the lifted Hamiltonian.

The resulting (Euclidean) Lagrangian is nonlocal and describes bosons which do and fermions which do not propagate through the lattice. All fields couple to the external magnetic field. The Lagrangian contains Yukawa and Luttinger type interactions.

1. Introduction: Motivation and Outline of the Approach

The isotropic $2D$ quantum Heisenberg antiferromagnet and the Hubbard model have received renewed attention in attempts to understand high T_c superconductivity. In particular this has been pursued by Anderson and his collaborators (see e.g. [A1, A2, ZA]) and by members of the Landau school (see [DPW, Pol, Wi1, Wi2]). In these last articles the aim is to exhibit the appearance of what the authors call Pomeranchuk fermions [Pom], which are supposed to be neutral spin 1/2 excitations describing the antiferromagnetic magnons of the theory. Similarly, the fermion solitons appearing in the resonating-valence bond theory (see e.g. [KRS]) are called spinons in [ZA]. In [DPW] and [Wi1, Wi2] (cf. also [FS]) the authors try to relate the quantum Heisenberg model via a Feynman-Kac formula to a $U(1)$-gauged CP_1-quantum field theory in $2D+1$ dimensions. The motivation is that the Pauli spin matrices are inappropriate operators when one tries to exhibit critical