Commun. Math. Phys. 131, 431–464 (1990)

Communications in Mathematical Physics

Variational Problems on Vector Bundles

Jürg Fröhlich and Michael Struwe

Departments of Mathematics and Physics, Eidgenössische Technische Hochschule, CH-8093 Zürich, Switzerland

Abstract. A variety of problems in quantum physics and classical statistical mechanics, in particular the quantization of topological solitons and the statistical mechanics of defects in ordered media, are described. These problems can be studied within a semi-classical approximation, or with the help of low-temperature expansions, respectively. The calculation of the leading term in such expansions gives rise to variational problems for sections of vector bundles characterized by certain topological constraints. Examples of such problems are the quantization of kinks in the two-dimensional $\lambda\varphi^4$-theory and the analysis of Bloch walls in a Landau-Ginzburg model of a three-dimensional anisotropic ferromagnet. We state a general existence result for variational problems of this kind and develop regularity and decay estimates for solutions of the Landau-Ginzburg model describing Bloch walls with prescribed boundaries. For certain boundary configurations stability results are established. The relation between the minimizers of the Landau-Ginzburg model in a certain strong-coupling limit and minimal surfaces is pursued in some detail. An open question is whether, asymptotically, the stability of the limit (minimal) surface will imply the stability of the minimizers of the Landau-Ginzburg model.

Contents

1. Background from Quantum Field Theory and Statistical Mechanics

The variational problems studied in this paper arise in the analysis of configurations of topological defects in ordered media, [1]. As a first example, we consider