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# The Newhouse Set Has a Positive Hausdorff Dimension

Xiao-Jing Wang*

Faculté des Sciences, Université Libre de Bruxelles, Campus Plaine, c.p. 231, Blvd du Triomphe, B-1050 Bruxelles, Belgium

**Abstract.** The Newhouse phenomenon of infinitely many coexisting periodic attractors is studied in its simplest form. One shows that the corresponding parameter set (the Newhouse set) $J_N$ has a strictly positive Hausdorff dimension. This result is stronger than that of Tedeschini-Lalli and Yorke [Commun. Math. Phys. **106**, 635 (1986)] concerning the Lebesgue measure of the Newhouse set; and is complementary to our knowledge on the topological properties of $J_N$, namely it is a residual set, hence uncountable and everywhere dense in a parameter interval.

## I. Introduction

It is nowadays well known, and much discussed in the literature, that in a multidimensional dynamical system, an infinite number of periodic *attractors* may coexist in a *bounded* region of the phase space. This possibility was recognized largely due to the work by Newhouse (1974, 1979). While studying the homoclinic tangency to a periodic saddle point in one-parameter families of planar diffeomorphisms, this author proved that the system near the tangency can possess an invariant basic set of chaotic nature, to which *secondary* homoclinic tangencies *persist* for an open set of parameter values. As a consequence, there is a residual parameter set (the Newhouse set), for which infinitely many sinks of arbitrarily long period coexist in the system.

One may naturally ask how typically does this phenomenon occur, or how generic is the Newhouse parameter set? We note immediately that there are two distinct basic notions of genericity for a set of real numbers: measure and category (Oxtoby, 1980). Being residual, the Newhouse set is generic in the topological sense of belonging to Baire's second category. It is not countable, and everywhere dense in an interval. The *measure* properties have been discussed by Tedeschini-Lalli and

* Current address: Mathematical Research Branch, NIDDK, Bldg 31, Room 4B-54, National Institutes of Health, Bethesda, MD 20892, USA