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Low Energy Bands do not Contribute to Quantum Hall Effect

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Abstract. Using non-commutative geometry and localization techniques we prove rigorously a result of Tesanovic, Axel and Halperin, namely that low energy bands in an ordered or slightly disordered 2D crystal submitted to a uniform magnetic field, do not contribute to the Hall conductivity.

1. Introduction

We first consider an electron gas in a perfect two dimensional crystal submitted to a perpendicular uniform magnetic field. If we neglect the Coulomb interaction between pairs of electrons and the motion of ions in the crystal, we are led to study the quantum motion of one charged particle (an electron or a hole) of mass m, of charge $\pm e$, in a periodic potential $\mu V(\mathbf{x})$ (here μ is a coupling constant) and a uniform magnetic field B.

For $\mu = 0$, the energy spectrum is given by the Landau levels $E_n = \hbar\omega_c(n + 1/2)$ where ω_c is the cyclotron frequency eB/m and $n = 0, 1, 2, \ldots$ [1]. The gap between E_{n-1} and E_n will be called G_n. It is well known that whenever the Fermi level is in G_n the quantum Hall conductance at zero temperature is quantized and equal to ne^2/h (h is the Planck constant). Both G_n and that quantization do survive for $\mu > 0$ small enough. This fact has been explained by recognizing that up to the universal physical constant e^2/h the Hall conductance at zero temperature is equal to the Chern class of the eigenprojection P_F of the Hamiltonian up to energies smaller than or equal to the Fermi level [2–8].

In a recent paper, B. Halperin et al. [9] have investigated through a numerical calculation how that scheme is modified as μ increases from zero to infinity for a genuine periodic potential V. For μ small, Landau levels broaden into bands B_n of width of order μ. Their structure is actually quite complicated for it is given in

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