

Asymptotic Mass Degeneracies in Conformal Field Theories

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Abstract. By applying a method of Hardy and Ramanujan to characters of rational conformal field theories, we find an asymptotic expansion for degeneracy of states in the limit of large mass which is *exact* for strings propagating in more than two uncompactified space-time dimensions. Moreover we explore how the rationality of the conformal theory is reflected in the degeneracy of states. We also consider the one loop partition function for strings, restricted to physical states, for arbitrary (irrational) conformal theories, and obtain an asymptotic expansion for it in the limit that the torus degenerates. This expansion depends only on the spectrum of (physical and unphysical) relevant operators in the theory. We see how rationality is consistent with the smoothness of mass degeneracies as a function of moduli.

Introduction

One of the characteristic features of strings is the prediction of the existence of infinitely many massive particles. Based on the fundamental principle of modular invariance (world-sheet reparametrization invariance) one deduces that the number of particles grows exponentially with mass. This growth is universal and independent of which background strings are propagating in. One of the simple consequences of this fundamental behaviour is the existence of a limiting temperature (the Hagedorn temperature) [1] beyond which the canonical partition function diverges.

For various reasons one might be interested in more detailed information about the large energy degeneracy of particle spectrum in string theory. Although the leading behaviour of degeneracies is independent of the background, as mentioned above, the subleading terms *do* depend on the background in which strings propagate. The subleading terms might be useful in computing certain thermodynamical quantities. In fact this point in connection with extending the thermodynamical computations in [2] was one of the main motivations for undertaking the present work. Another application of these subleading terms might be in probing high energy aspects of string theory. Finally, these expansions might