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New Results on the Moments of the Eigenvalues of the Schrödinger Hamiltonian and Applications

A. Martin

Theory Division, CERN, CH-1211 Geneva 23, Switzerland

Abstract. We show that the moments of order γ of the eigenvalues of the Schrödinger Hamiltonian in n dimensions can be related to moments of order less than or equal to $\gamma-\frac{1}{2}$ in $n+1$ dimensions. This makes it possible to improve the bounds on the sum of the eigenvalues in three dimensions and consequently the Lieb-Thirring bound on the binding energy of matter.

1. Introduction

A technical, but essential ingredient of the proof of "stability" of matter by Lieb and Thirring [1] is a bound on the sum of the eigenvalues of a one-particle Hamiltonian in three space dimensions. The Lieb-Thirring bound has the form

$$E>-CN\left[1+\left(\frac{\sum_{j=1}^{p} Z_j^{7/3}}{N}\right)^{1/2}\right]^2 \alpha^2 m \tag{1}$$

for a system with N electrons and point nuclei of charges $Z_1, Z_2, \ldots, Z_p$; α is the fine structure constant and m the electron mass.

The constant C is proportional to the power 2/3 of the coefficient appearing in the inequality

$$\sum |e_i| < L_{1,3} \int |V^-|^{5/2} d^3x, \tag{2}$$

where the e_i's are the negative eigenvalues of the three-dimensional one-particle Schrödinger equation with a potential V (for the notation $L_{1,3}$ see Sect. 2). In [1], Lieb and Thirring get

$$L_{1,3} \leqq \frac{4}{15\pi}, \tag{3}$$

and correspondingly

$$C=4.16. \tag{4}$$