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Slowly Rotating Drops

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Abstract. We study the existence of gyrostatic equilibria of slowly rotating liquid masses subject only to the force of surface tension. We give a rigorous proof of the existence of non-axisymmetric equilibria. The shape of such an equilibrium approximates a number of spherical lobes connected by thin necks and symmetrically arranged around the axis of rotation.

1. Introduction

The study of the gyrostatic equilibria of rotating liquid masses and their stability is an old problem in Mathematical Physics. The subject probably starts with Newton's discussion of the shape of the Earth in 1687. He modeled the Earth as a homogeneous gravitating liquid and he proposed the figure of an oblate spheroid as a gyrostatic equilibrium. The subject was developed by Maclaurin (1740), Jacobi (1843), and Poincaré (1885) who studied more complicated equilibria and their stability.

In this century Lyttleton [Ly] and Chandrasekhar [Ch] made further contributions. Also models with more forces were allowed, for example surface tension, electrostatic forces produced by uniformly distributed electric charge, and others. This gave crude models for heavy nuclei [B–W]. The interested reader is referred to [Sw, C–P–S, Ly, and Ch], where he can find further references.

We are interested in the case when the only non-negligible force involved (besides friction) is the surface tension. This subject started with the experiments of J. A. F. Plateau [Pl] who, although blind, produced an impressive amount of work. To our knowledge, the most interesting recent work in this field is a numerical study of the shapes and stability of rotating drops by Brown and Scriven [B–S], and to which we will often refer in this paper. Experimental work in the space shuttle has been carried out [W–T–C–E]. Tsamopoulos and Brown [T–B] have further studied stability questions. The case of planar drops has also been studied, see [B, L–M–R], where further references can be found.

We limit ourselves in this paper to studying slowly rotating drops, that is gyrostatic equilibria which would be plotted close to the origin in Figs. 3 or 4 of