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The Time Dependent Correlation Function of an Impenetrable Bose Gas as a Fredholm Minor. I

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Abstract. We study the two field correlator of an Impenetrable Bose gas. Lenard [1] proved that the equal time correlator can be represented as a Fredholm minor. We generalize this representation to the time dependent correlator.

1. Introduction

We discuss the Bose gas in one space plus one time dimensions. The Hamiltonian [2] of the model is

$$H = \int_{-L/2}^{L/2} dx(\partial_x \psi^+ \partial_x \psi + c\psi^+ \psi^+ \psi\psi - h\psi^+ \psi). \tag{1.1}$$

Here $\psi(x)$ and $\psi^+(x)$ are canonical Bose fields: $[\psi(x), \psi^+(y)] = \delta(x-y)$, and L is the volume. Only the case of an impenetrable boson is dealt with below, in this case the coupling constant $c = \infty$. The thermodynamics of the Bose gas was given in [3]. The chemical potential h determines uniquely the density D. In [1] Lenard gives a representation of the time independent correlator $\langle \psi(x_2)\psi^+(x_1)\rangle$ as a Fredholm minor; this representation was used to write a differential equation for the correlator at zero temperature in [4]. The differential equation for the finite-temperature correlator was constructed in [5]. We can treat the Fredholm determinant obtained in this paper as a Gelfand–Levitan operator for some new differential equation, which describes the time dependent field correlator of the impenetrable Bose gas. The correlation function (Fredholm determinant) is the τ-function of this new differential equation. We shall present this differential equation in the next publication.

Eigenfunctions of the Hamiltonian (1.1) (at $c = \infty$) were constructed in [2].

$$H|\Psi_N\rangle = E|\Psi_N\rangle; \quad |\Psi_N\rangle = |\Psi_N(\{\lambda\})\rangle, \tag{1.2}$$

$$|\Psi_N\rangle = \frac{1}{\sqrt{N!}} \int d^N z \chi_N(z_1, \ldots, z_N|\lambda_1, \ldots, \lambda_N)\psi^+(z_1)\cdots\psi^+(z_N)|0\rangle. \tag{1.3}$$