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Orbifolds as Configuration Spaces of Systems with Gauge Symmetries

C. Emmrich and H. Römer
Fakultät für Physik der Universität Freiburg, Hermann-Herder-Straße 3, D-7800 Freiburg, FRG

Abstract. In systems like Yang–Mills or gravity theory, which have a symmetry of gauge type, neither phase space nor configuration space is a manifold but rather an orbifold with singular points corresponding to classical states of non-generically higher symmetry. The consequences of these symmetries for quantum theory are investigated. First, a certain orbifold configuration space is identified. Then, the Schrödinger equation on this orbifold is considered. As a typical case, the Schrödinger equation on (double) cones over Riemannian manifolds is discussed in detail as a problem of selfadjoint extensions. A marked tendency towards concentration of the wave function around the singular points in configuration space is observed, which generically even reflects itself in the existence of additional bound states and can be interpreted as a quantum mechanism of symmetry enhancement.

1. Introduction

Let a Lie group $\mathbf{G}$ act on a manifold M. Then it is natural to perform a "symmetry reduction" by identifying points which can be transformed into one another and, hence, going over to the space $M/\mathbf{G}$, the set of orbits in M under the action of $\mathbf{G}$. Now, $M/\mathbf{G}$ will, in general, not be a manifold, but rather an orbifold. Singular points in $M/\mathbf{G}$ will arise whenever a jump occurs in the conjugacy class $(\mathbf{H})$ of the isotropy groups $\mathbf{H} \subset \mathbf{G}$ of points in different orbits.

In this note, we shall investigate the impact of such orbifold singularities in configuration and phase space of mechanical systems to the associated quantum systems, thus expanding and elaborating on a programme announced in [1].

This introduction contains a description of the main ideas and results, all technical details will be deferred to following sections. Our framework will be Lagrangian mechanics (and field theory) with a (possibly infinite dimensional) manifold Q as configuration space and its cotangent bundle $P = T^*Q$ as phase space. Let now $\mathbf{G}$ act on Q. For every $\xi \in \mathscr{G}$, the Lie algebra of $\mathbf{G}$, there is a fundamental vector field ξ_Q on Q, defined by