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# Homological Perturbation Theory and the Algebraic Structure of the Antifield-Antibracket Formalism for Gauge Theories

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**Abstract.** The algebraic structure of the antifield-antibracket formalism for both reducible and irreducible gauge theories is clarified. This is done by using the methods of Homological Perturbation Theory (HPT). A crucial ingredient of the construction is the Koszul-Tate complex associated with the stationary surface of the classical extremals. The Koszul-Tate differential acts on the antifields and is graded by the antighost number. It provides a resolution of the algebra $\mathscr{A}$ of functions defined on the stationary surface, namely, it is acyclic except at degree zero where its homology group reduces to $\mathscr{A}$. Acyclicity only holds because of the introduction of the ghosts of ghosts and provides an alternative criterion for what is meant by a proper solution of the master equation. The existence of the BRST symmetry follows from the techniques of HPT. The classical Lagrangian BRST cohomology is completely worked out and shown to be isomorphic with the cohomology of the exterior derivative along the gauge orbits on the stationary surface. The algebraic structure of the formalism is identical with the structure of the Hamiltonian BRST construction. The role played there by the constraint surface is played here by the stationary surface. Only elementary quantum questions (general properties of the measure) are addressed.

## 1. Introduction

Over the past few years it has become clear that the BRST charge plays a key role in field theory. Recently, the algebraic structure of the classical BRST symmetry has been elucidated in the Hamiltonian formalism for the general case of reducible theories [1]. The crucial point there was the construction of the "Koszul-Tate" differential operator $\delta$ associated with the constraint surface. With this operator at hand, the algebraic properties of the classical BRST charge in the Hamiltonian

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