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Random Hamiltonians Ergodic in All But One Direction

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Abstract. Let $V_\omega^{(1)}$ and $V_\omega^{(2)}$ be two ergodic random potentials on $\mathbb{R}^d$. We consider the Schrödinger operator $H_\omega = H_0 + V_\omega$, with $H_0 = -\Delta$ and for $x = (x_1, \ldots, x_d)$

$$V_\omega(x) = \begin{cases} V_\omega^{(1)}(x) & \text{if} \quad x_1 < 0 \\ V_\omega^{(2)}(x) & \text{if} \quad x_1 \geqq 0 \end{cases} .$$

We prove certain ergodic properties of the spectrum for this model, and express the integrated density of states in terms of the density of states of the stationary potentials $V_\omega^{(1)}$ and $V_\omega^{(2)}$. Finally we prove the existence of the density of surface states for H_ω.

1. Introduction

In this paper we consider Schrödinger operators $H_\omega = H_0 + V_\omega$ with random potential V_ω on $L^2(\mathbb{R}^d)$. The random potential V_ω we consider has different behavior in the left and right half space. More precisely, there are two ergodic random fields V_ω^+ and V_ω^- on $\mathbb{R}^d$ such that V_ω agrees with V_ω^+ in one half space and with V_ω^- in the complementary half space. To be specific we assume $V_\omega(x) = V_\omega^+$ for $x_1 \geqq 0$ and $V_\omega(x) = V_\omega^-(x)$ for $x_1 < 0$.

Thus V_ω is not an ergodic potential (unless $V_\omega^\pm$ happen to agree). Consequently, the general theory of ergodic potentials (see e.g. [4, 2, 10] and references therein) does not apply. For example, a priori the spectrum $\sigma(H_\omega)$ may depend on ω. In fact, Molcanov and Seidel [15] consider the one dimensional case in detail. They prove that, in their special case, the spectrum $\sigma(H_\omega)$ consists of the half line $[0, \infty)$ plus an additional isolated negative eigenvalue. This eigenvalue depends on the random parameters.

We will prove in the next section that in the higher dimensional case ($d > 1$) the spectrum is non-random under very mild assumptions. The main difference between $d = 1$ and $d > 1$ lies in the "ergodicity" of the potential under shifts parallel