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Isoholonomic Problems and Some Applications

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Abstract. We study the problem of finding the shortest loops with a given holonomy. We show that the solutions are the trajectories of particles in Yang–Mills potentials (Theorem 4), or, equivalently, the projections of Kaluza–Klein geodesics (Theorem 2). Applications to quantum mechanics (Berry's phase, Sect. 3) and the optimal control of deformable bodies (Sect. 6) are touched upon.

Contents

1. The Problem and an Introduction

1.1 The Problem which we investigate is *the isoholonomic problem*: **among all loops with a fixed holonomy, find the loop of minimum length.**

The data needed to formulate this problem are a principal bundle

$$\pi : Q \to X \qquad [1.1]$$

with connection A, a Riemannian metric k on X, and a point $x_0 \in X$ at which the loop and its holonomy are based. (The holonomy is called the Wilson loop integral, or the path-ordered exponential of $-A$ in the physics literature.) The structure group of the bundle will be denoted by G. It is a Lie group which acts on Q on the right, and such that $X \cong Q/G$.