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Dynamics of Fluctuations for Quantum Lattice Systems

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Abstract. For short range interactions and for L^1-space clustering states it is proved that there exists a bonafide time evolution on the set of normal fluctuations. This dynamics is applied to derive the notion of equilibrium state of the algebra of fluctuations.

1. Introduction

The aim of statistical mechanics is the explanation of the macrophenomena on the basis of the microstructure. For this it is important to make a clear distinction between microscopic observables, states, etc. and macroscopic ones.

In the algebraic approach to quantum statistical mechanics the microscopic system is currently described by a (C^*- or von Neumann-) algebra $\mathscr{B}$ of observables. A physical state is a positive linear normalized functional ω of $\mathscr{B}$ and a dynamics of the system $(\mathscr{B},\omega)$ is a one-parameter group α_t of $*$-automorphisms of $\mathscr{B}$.

Analogously any macroscopic system should be described by an analogous triplet of observables, states and dynamics.

The problem is to construct by precise procedures the macroscopic triplets out of the microscopic ones.

A well known [1] algebra of macroscopic observables is the one given by the observables at infinity and consisting of the space means of local micro-observables: i.e. for A any local observable, one considers the corresponding observable at infinity,

$$A_\omega^\infty = \lim_{V\to\infty} \frac{1}{V}\int_V dx\,\tau_x A,$$

where V is any finite volume and τ_x the translation over the space variable x. In probability this limit is known as the law of large numbers. The algebra generated by the limits $\{A_\omega^\infty \mid A\in\mathscr{B}\}$ is an abelian algebra of macroscopic observables.

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