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Quantum 2-Spheres and Big q-Jacobi Polynomials

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Abstract. Orthogonal bases for the algebras of functions of Podles' quantum 2-spheres are explicitly determined in terms of big q-Jacobi polynomials. This gives a group-theoretic interpretation of the symmetric big q-Jacobi polynomials and the symmetric q-Hahn polynomials.

Quantum groups, introduced by Drinfeld [D], Jimbo [J] and Woronowicz [W1], are now realized to provide a good framework for q-analogues of special functions. The little q-Jacobi polynomials were the first example of q-orthogonal polynomials to be understood by quantum groups. It was found by Vaksman–Soibelman [VS], Masuda et al. [M0] and Koornwinder [K1]that they naturally appear as matrix elements of the irreducible unitary representations of the quantum group $SU_q(2)$ (see also [K0] and [M1]). Up to now, it is also known by Kirillov–Reshetikhin [KR] and Koelink–Koornwinder [KK] that the Clebsch–Gordan coefficients for $SU_q(2)$ are expressed in terms of the q-Hahn polynomials.

In this paper, we will show that the *big q-Jacobi polynomials* $P_n^{(\alpha,\alpha)}(x;c,d;q)$ *of symmetric type* appear as spherical functions on the *quantum 2-spheres of Podles.* Quantum 2-spheres are studied by P. Podles [P] from the viewpoint of operator algebra theory. He also gives the irreducible decomposition of their algebras of functions. We will determine explicitly their orthogonal bases in terms of the big q-Jacobi polynomials.

Throughout this paper, we denote by G the quantum group $SU_q(2)$, where q is a *real number* with $0 < q < 1$. The algebra of functions $A(G)$ is a Hopf algebra over $\mathbb{C}$ with a $*$-operation. As for quantum groups, we will follow the notation and the terminology of [M1].

Podles' quantum 2-spheres are a family of quantum G-spaces. A *quantum G-space X* is a quantum space "on which the quantum group G acts." This means that the algebra of functions $A(X)$ on X has the structure of a left (or right) $A(G)$-comodule such that the structure mapping $L_G: A(X) \to A(G) \otimes_{\mathbb{C}} A(X)$ is a $\mathbb{C}$-algebra homomorphism. (When we consider the real structure of X, we also require that $A(X)$ has a $*$-operation and that L_G is compatible with the $*$-structure.)